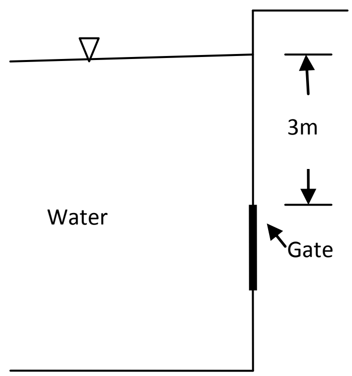
Tutorial 3
Hydrostatic force on submerged bodies

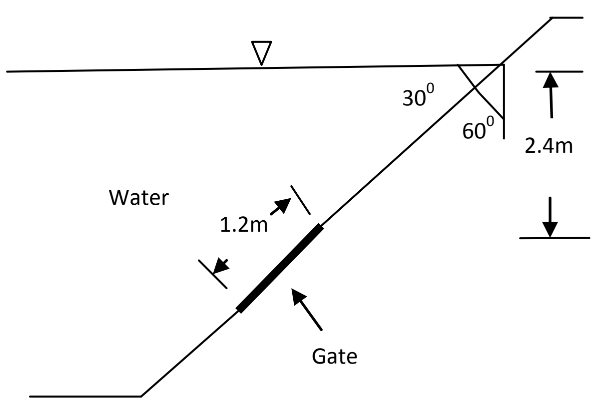
1. A vertical rectangular gate, 1.4m high and 2 m wide, contains water on one side. Determine the total resultant force acting on the gate and the location of c.p.



Solution:
Area (A) = 2x1.4 = 2.8 m^2
Location of CG ($\bar{y}$) = (3+1.4/2) = 3.7m
Resultant force on gate (F) = ?
Cp (y_p) = ?

$F = \gamma A\bar{y}$ = 9810x2.8x3.7 = 101631 N = 101.631 KN
M.I. about CG $(I_G) = \frac{1}{12}x2x1.4^3$ = 0.457 m^4
$y_p = \bar{y} + \frac{I_G}{A\bar{y}} = 3.7 + \frac{0.457}{2.8x3.7}$ =3.74m

2. An inclined rectangular gate (1.5m wide) contains water on one side. Determine the total resultant force acting on the gate and the location of c.p.

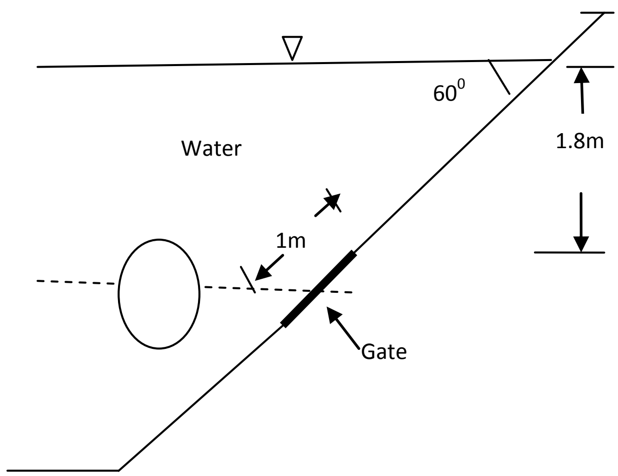


Solution:

Area (A) = 1.5x1.2 = 1.8 m^2
Location of CG ($\bar{y}$) = (2.4+1.2Sin30/2) = 2.7m
Resultant force on gate (F) = ?
Cp (y_p) = ?

$F = \gamma A\bar{y}$ = 9810x1.8x 2.7 = 47676N = 47.676 KN
M.I. about CG $(I_G) = \frac{1}{12}x1.5x1.2^3$ = 0.216 m^4
$y_p = \bar{y} + \frac{I_G Sin^2\theta}{A\bar{y}} = 2.7 + \frac{0.216Sin^2 30}{1.8x2.7}$ = 2.71m

3. An inclined circular with water on one side is shown in the fig. Determine the total resultant force acting on the gate and the location of c.p.

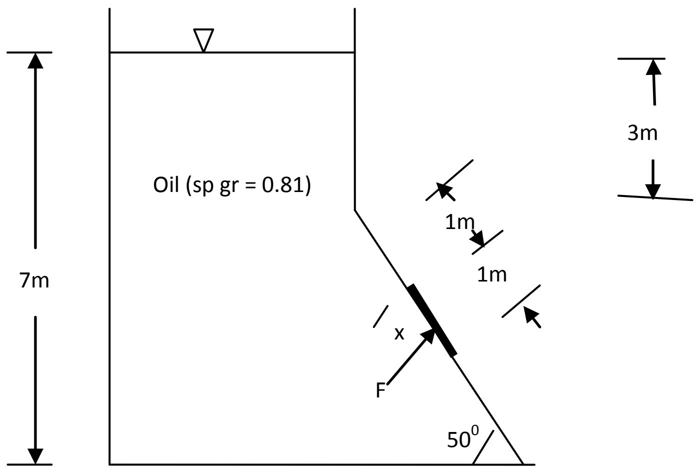


Solution:

Area (A) = $\pi x \frac{1^2}{4}$ = 0.785 m^2
Location of CG ($\bar{y}$) = (1.8+1.0Sin60/2) = 2.23m
Resultant force on gate (F) = ?
Cp (y_p) = ?

$F = \gamma A\bar{y}$ = 9810x0.785x2.23 = 17173N = 17.173 KN
M.I. about CG $(I_G) = \frac{\pi}{64}x1^4$ = 0.049 m^4
$y_p = \bar{y} + \frac{I_G Sin^2\theta}{A\bar{y}} = 2.23 + \frac{0.049Sin^2 60}{0.785x2.23}$ = 2.25m

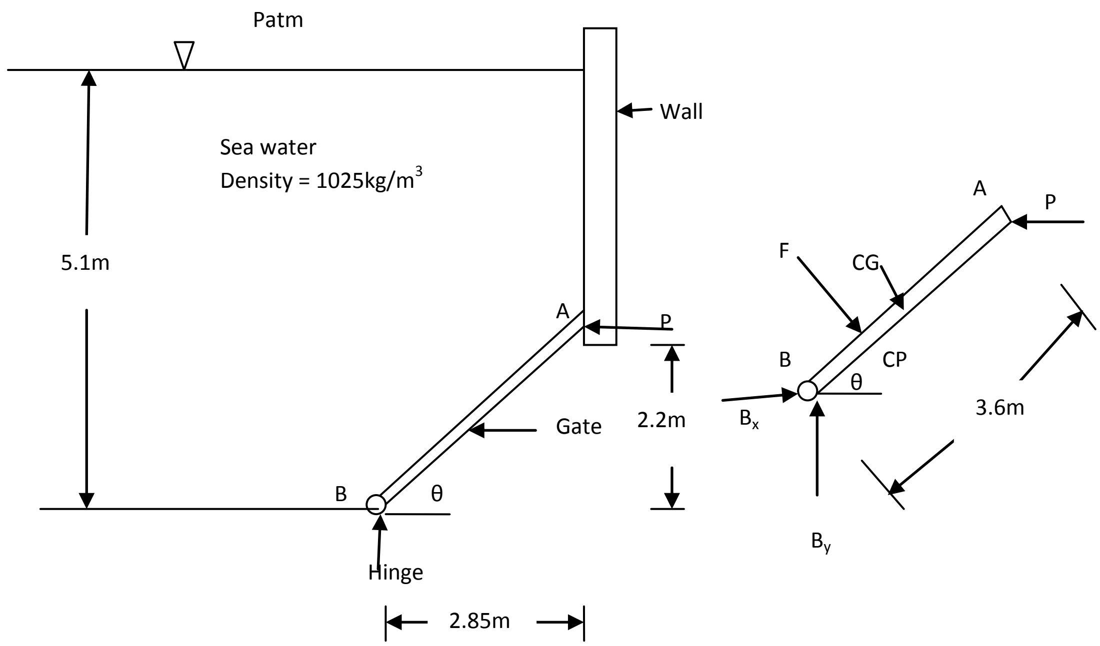
4. Gate AB in the fig. is 1m long and 0.7m wide. Calculate force F on the gate and position X of c.p.



Solution:
Sp. wt of oil (γ) =0.81x9810 = 7946 N/m3
Area (A) = 0.7x1 = 0.7 m^2
Location of CG ($\bar{y}$) = (3+1Sin50+1Sin50/2) = 4.15m
Resultant force on gate (F) = ?
x = ?

$F = \gamma A\bar{y}$ = 7946x0.7x4.15 = 23083 N = 23.08 KN
M.I. about CG $(I_G) = \frac{1}{12}x0.7x1^3$ = 0.058 m^4
Vertical distance of CP from free surface
$y_p = \bar{y} + \frac{I_G Sin^2\theta}{A\bar{y}} = 4.15 + \frac{0.058Sin^2 50}{0.7x4.15}$ = 4.161m
Vertical distance between CP from CG = 4.161-(3+1Sin50)= 0.395m
x = 0.395/sin50= 0.515m

5. The gate in the fig. is 1.2m wide, is hinged at point B, and rests against a smooth wall at A. Compute (a) the force on the gate due to sea water pressure, (b) the horizontal force exerted by the wall at point A, and (c) the reaction at hinge B.



Solution:

Sp wt of sea water (γ) = 1025x9.81 = 10055 N/m^3

Area (A) = 1.2x3.6 = 4.32 m^2

Location of CG ($\bar{y}$) = (5.1-2.2)+2.2/2 = 4.0m

a) Resultant force on gate (F) = ?

$F = \gamma A\bar{y}$ = 10055x4.32x4.0 = 173750 N = 173.75 KN

b) Force P = ?

M.I. about CG $(I_G) = \frac{1}{12}x1.2x3.6^3$ = 4.665 m^4

Vertical distance of CP from free surface

$y_p = \bar{y} + \frac{I_G Sin^2\theta}{A\bar{y}} = 4.0 + \frac{4.665x(2.2/3.6)^2}{4.32x4}$ = 4.1m

Vertical distance between B and CP = 5.1-4.1 = 1m

Location of F from B = 1/sinθ = 1.636m

Taking moment about B,

Px2.2-173.75x1.636 = 0

P = 129.2KN

c) Reactions at hinge, B_x and B_y = ?

$\sum F_x = 0$

B_x – 129.2 + 173.75x2.2/3.6 = 0

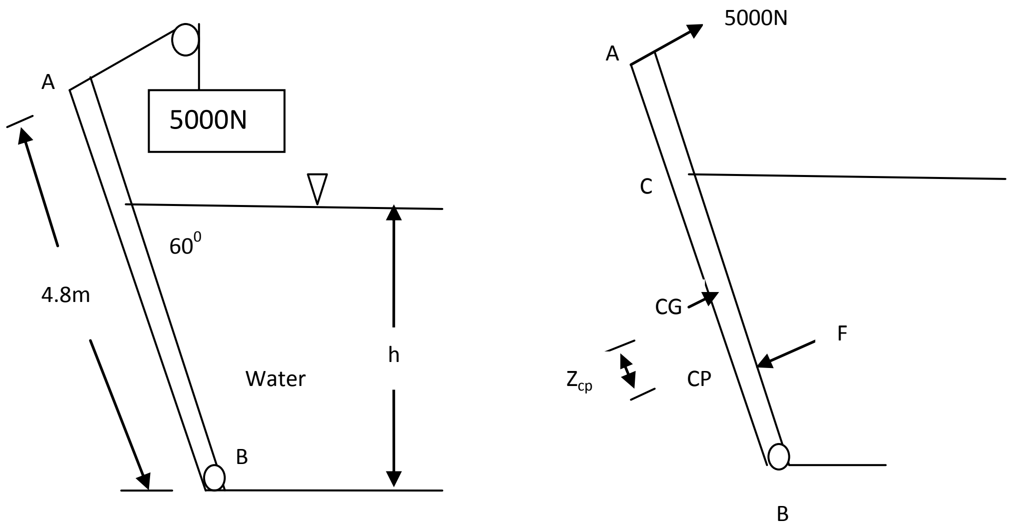
B_x = 23.02 KN

$\sum F_y = 0$

B_y - 173.75x2.85/3.6 = 0

B_y = 137.55 KN

6. Gate AB in fig. is 4.8m long and 2.4m wide. Neglecting the weight of the gate, compute the water level h for which the gate will start to fall.



Solution:

Area (A) = 2.4xh/Sin60 = 2.77h m^2

Location of CG ($\bar{y}$) = h/2 m

Resultant force on gate (F) is

$F = \gamma A\bar{y}$ = 9810x2.77hxh/2 = 13587h^2 N

M.I. about CG $(I_G) = \frac{1}{12}x2.4x(\frac{h}{sin60})^3$ = 0.308h^3 m^4

Vertical distance of CP from free surface

$y_p = \bar{y} + \frac{I_G Sin^2\theta}{A\bar{y}} = h/2 + \frac{0.308h^3\ Sin^2 60}{2.77hxh/2}$ = 0.667h

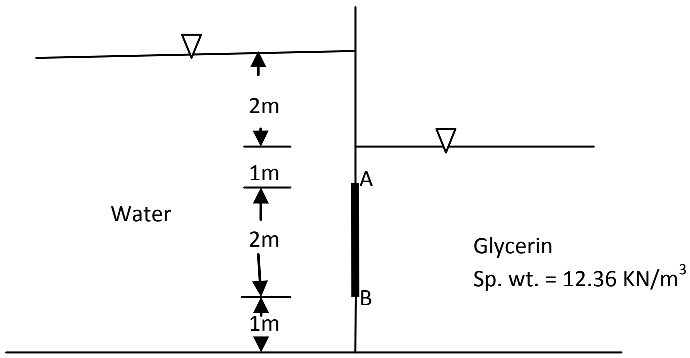
Distance of F from B = (h-0.667h)/Sin60=0.384h

Taking moment about B,

5000x4.8-13587h^2x0.384h = 0

h = 1.66m

7. Find the net hydrostatic force per unit width on rectangular panel AB in the fig. and determine its line of action.

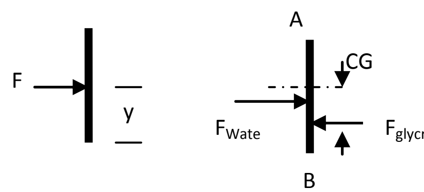


Solution:

Area (A) = 2x1 = 2 m^2

Location of CG ($\bar{y}$) = 2+1+2/2 = 4 m for water side

Location of CG ($\overline{y1}$) = 1+2/2 = 2 m for glycerin side



Resultant force on gate (F) = ?

$F = \gamma A\bar{y}$

Force due to water (F_{water}) = $\gamma A\bar{y}$ = 9.81x2x4 = 78.49KN

Force due to glycerin (F_{glyc}) = $\gamma A\overline{y1}$ =12.36x2x2 = 49.44KN

Net force (F) = 78.49-49.44 = 29.04KN

M.I. about CG $(I_G) = \frac{1}{12}x1x2^3$ = 0.666 m^4

Distance of F_{water} from CG $(y_{cp1}) = \bar{y} + \frac{I_G}{A\bar{y}} = 4 + \frac{0.666}{2x4}$ = 4.083m

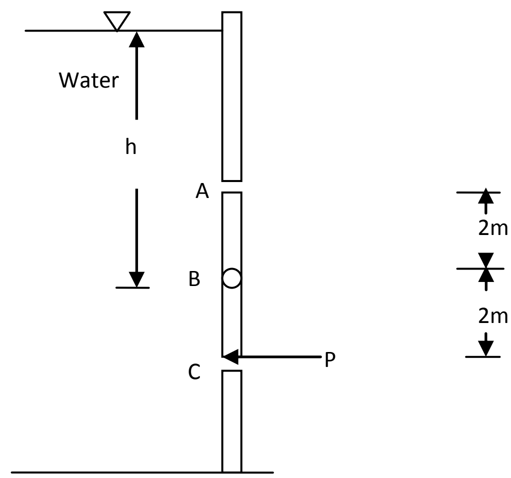
Distance of F_{glyc} from CG $(y_{cp2}) = \bar{y} + \frac{I_G}{A\overline{y1}} = 2 + \frac{0.666}{2x2}$ = 2.166m

Taking moment about B,

29.04y = 78.49x(5-4.083)-49.44x(3-2.166)

y = 0.945m

8. Circular gate ABC in the fig. is 4m in diameter and is hinged at B. Compute the force P just sufficient to keep the gate from opening when h is 8m.

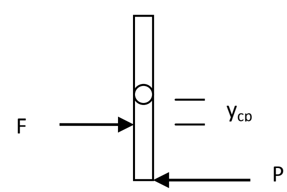


Solution:

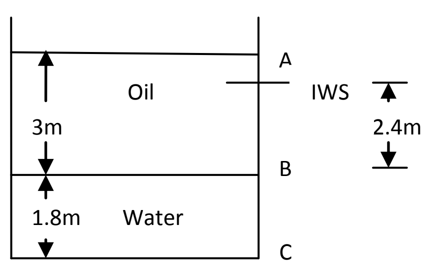
Area (A) = $\pi x \frac{4^2}{4}$ = 12.56 m^2
Location of CG ($\bar{y}$) = 8m
P = ?
Resultant force on gate (F)
$F = \gamma A\bar{y}$ = 9810x12.56x8 = 985708 N = 985.708KN
M.I. about CG $(I_G) = \frac{\pi}{64} x 4^4$ = 12.56 m^4
Position of CP from free surface
$y_{cp} = \bar{y} + \frac{I_G}{A\bar{y}} = 8 + \frac{12.56}{12.56x8}$ = 8.125 m
Taking moment about B,

985.7x0.125-Px2=0

P = 61.6KN



9. The tank in the fig. contains oil (sp gr = 0.8) and water as shown. Find the resultant force on side ABC and its point of application. ABC is 1.2m wide.



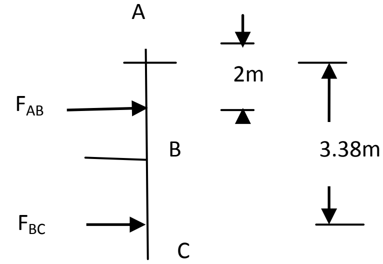
Solution:

Sp wt of oil (γ_{oil}) = 0.8*9810 N/m^3 = 7848 N/m^3
Area (A1) = 1.2x3 = 3.6 m^2

Area (A2) = 1.2x1.8 = 2.16 m^2
Location of CG for AB ($\overline{y1}$) = 3/2 =1.5m
Resultant force on ABC = ?

Force on AB
$F_{AB} = \gamma_{oil} A1\overline{y1}$ =7848x3.6x1.5 = 42379N
Pt. of application of F_{AB} = (2/3)x3 = 2m below A

Force on BC
Water is acting on BC and any superimposed liquid can be converted to an equivalent depth of water.
Equivalent depth of water for 3m of oil = $\frac{\gamma_{oil}\, h_{oil}}{\gamma} = \frac{7848x3}{9810}$ = 2.4m
Employ an imaginary water surface of 2.4m.
Location of CG for IWS ($\overline{y2}$) = 2.4+1.8/2 = 3.3m



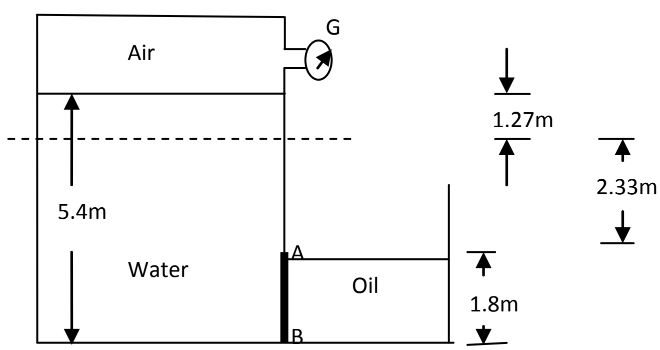
$F_{BC} = \gamma A2\overline{y2}$ = 9810x2.16x3.3 = 69925N
Point of application of F_{BC} from A is

$\bar{h} = \overline{y2} + \frac{I_G}{A\overline{y2}} = 3.3 + \frac{\frac{1}{12}x1.2x1.8^3}{2.16x3.3}$ =3.38 m
i.e. 3.38+0.6 = 3.98m from A

Total force on side ABC (F) = 42379+69925 = 112304N = 112.304 KN
Taking moment about A,
112304 y = 42379x2+69925x3.98
y = 3.23m
F acts at 3.23m below A.

(Alternative method: Solve by drawing pressure diagram. Force = Area of pressure diagram x width. Take moment to find position of resultant force.)

10. Gate AB in the fig. is 1.25m wide and hinged at A. Gage G reads -12.5KN/m^2, while oil (sp gr = 0.75) is in the right tank. What horizontal force must be applied at B for equilibrium of gate AB?



Solution:
Sp wt of oil (γ_{oil}) = 0.75*9810 N/m^3 = 7357.5 N/m^3
Area (A) = 1.25x1.8 = 2.25 m^2

Location of CG for right side ($\overline{y1}$) = 1.8/2 =0.9m

Force on AB at the right side

$F_{oil} = \gamma_{oil} A\overline{y1}$ =7357.5x2.25x0.9 = 14899N

Pt. of application of F_{oil} = (2/3)x1.8 = 1.2m from A

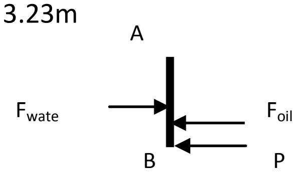
Left side

For the left side, convert the negative pressure due to air to equivalent head in water.

Equivalent depth of water for -12.5KN/m^2 pressure = $\frac{P}{\gamma} = \frac{-12.5}{9.81}$ = -1.27m

This negative pressure head is equivalent to having 1.27m less water above A.

Location of CG from imaginary water surface ($\overline{y2}$) = 2.33 + 1.8/2 = 3.23m



$F_{water} = \gamma A\overline{y2}$ = 9810x2.25x3.23 = 71294N

Point of application of F_{water} from A is

$\bar{h} = \bar{y} + \frac{I_G}{A\bar{y}} = 3.23 + \frac{\frac{1}{12}x1.25x1.8^3}{2.25x3.23}$ =3.31m from IWS

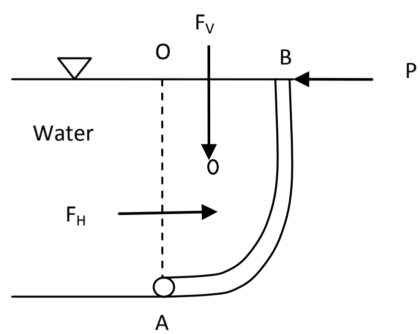
i.e. 3.31-2.33 = 0.98m from A

Taking moment about A

Px1.8 + 14899x1.2-71294x0.98 = 0

P = 28883 N = 28.883 KN

11. The gate AB shown is hinged at A and is in the form of quarter-circle wall of radius 12m. If the width of the gate is 30m, calculate the force required P to hold the gate in position.



Solution:

Horizontal force $(F_H) = \gamma A\bar{y}$ = 9810x(30x12)x12/2 = 21189600 N = 21189.6 KN (right)

F_H acts at a distance of 12x1/3 =4m above the hinge A.

Vertical force (F_v) = Weight of volume of water vertically above AB = $\gamma\ Volume_{AOB}$

$= 9810x\left[\frac{\pi x12^2}{4}x30\right]$ = 33284546 N = 33284.546 KN (downward)

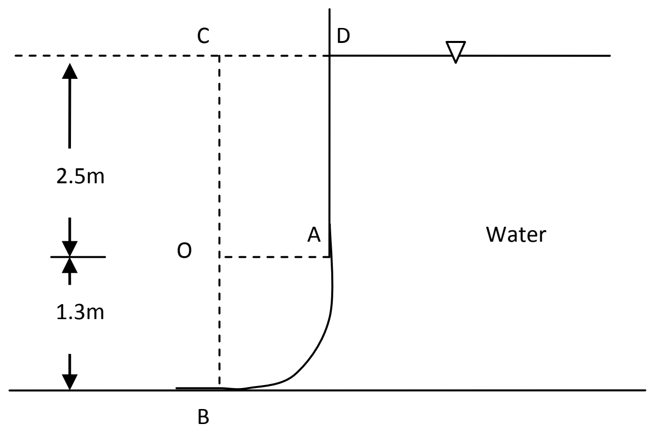
F_V acts at a distance of $4r/3\pi$ = 4x12/3x3.1416 = 5.1m from the vertical AO.

Taking moment about A,

Px12= 21189.6x4+33284.546x5.1

P = 21209 KN

12. The water is on the right side of the curved surface AB, which is one quarter of a circle of radius 1.3m. The tank's length is 2.1m. Find the horizontal and vertical component of the hydrostatic acting on the curved surface.



Solution:

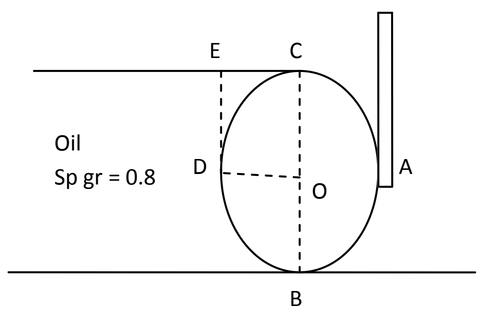
Horizontal force $(F_H) = \gamma A\bar{y}$ = 9810x(1.3x2.1)x(2.5+1.3/2) = 84361 N = 84.361 KN (right)

Vertical force (Fv) = Weight of imaginary volume of water vertically above AB

= $\gamma\ [Volume_{AOB} + Volume_{AOCD}]$

$= 9810x\left[\frac{\pi x 1.3^2}{4} x2.1 + 2.5x1.3x2.1\right]$ = 94297 N = 94.297KN (downward)

13. The 1.8m diameter cylinder in the fig. weighs 100000N and 1.5m long. Determine the reactions at A and B, neglecting friction.



Solution:

Horizontal force $(F_H) = \gamma A\bar{y}$ = 0.8x9810x(1.8x1.5)x(1.8/2) = 19071 N (right)

Vertical force (F_v) = Weight of volume of water vertically above BDC

Vertical force (F_v) =$(F_v)_{DB}$ - $(F_v)_{DC}$ = $\gamma\ [Volume_{BDECO} - Volume_{DECD}]$

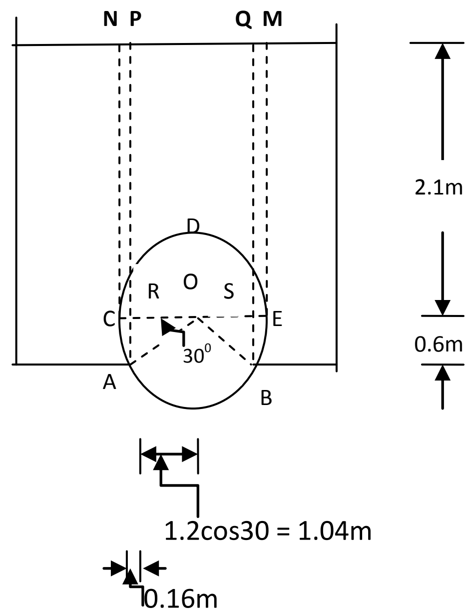
$= \gamma Volume_{BCD}$

$=0.8x9810x\left[\frac{\pi x 0.9^2}{2} x1.5\right]$ = 14978 N (up)

Reaction at A = F_H = 19071 N (left)

Reaction at B = Weight of cylinder – F_V = 100000-14978 = 85022N (up)

14. In the fig., a 2.4m diameter cylinder plugs a rectangular hole in a tank that is 1.4m long. With what force is the cylinder pressed against the bottom of the tank due to the 2.7m depth of water?



Solution:

Water is above the curve portion CDE, whereas it is below the curve portion AC and BE. For AC and BE, imaginary weight of water vertically above them is considered and the vertical force on theses part acts upwards.

Net vertical force= $(F_V)_{CDE}$ (down) – $(F_V)_{AC}$ (up) – $(F_V)_{BE}$ (up)

= Weight of volume of water vertically above CDE- imaginary Weight of volume above arc AC- imaginary weight of volume above arc BE

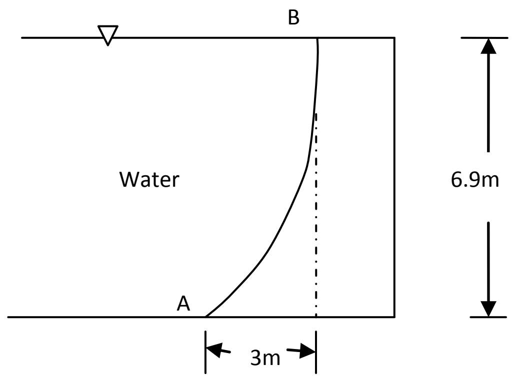
$=\gamma\ [Volume_{\ above\ CDE} - Volume_{\ above\ AC} - Volume_{\ above\ BB}]$

$$= 9810x[(volume\ rectangle\ CEMN - volume\ semicircle\ CDE)$$
$$- (volume\ rectangleCRPN + volume\ sectorCOA - volume\ triangleROA)$$
$$- (volume\ rectangleSEMQ + volume\ sectorBOE - volume\ triangleBOS)]$$

$=9810x\left[\left(2.4x2.1x1.4 - \frac{\pi x 1.2^2}{2} x1.4\right) - \left(0.16x2.1x1.4 + \frac{30}{360} x\pi x1.2^2 x1.4 - \frac{1}{2} 1.04x.6x1.4\right) - \left(0.16x2.1x1.4 + \frac{30}{360} \pi x1.2^2 x1.4 - \frac{1}{2} 1.04x.6x1.4\right)\right]$

= 27139N (down)

15. A dam has a parabolic profile as shown in the fig. Compute the horizontal and vertical components of the force on the dam due to the water. The width of dam is 15m. (Parabolic area = 2/3(b*d))



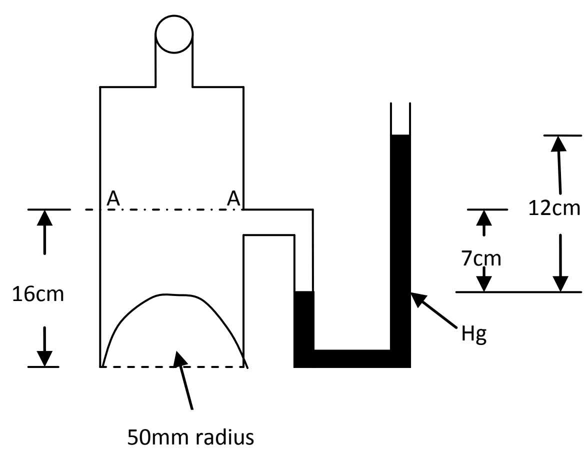
Solution:

Horizontal force $(F_H) = \gamma A\bar{y}$ = 9810x(15x6.9)x6.9/2 = 3502906 N = 3502.906 KN (right)

Vertical force (Fv) = Weight of volume of water vertically above AB = $\gamma\ Volume_{\ above\ AB}$

$= 9810\ x\frac{2}{3}x3x6.9x15$ = 2030670 N = 2030.67 KN (down)

16. The bottled liquid (sp gr = 0.9) in the fig. is under pressure, as shown by the manometer reading. Compute the net force on the 50mm radius concavity in the bottom of the bottle.



Solution:

From symmetry, F_H = 0

Manometeric equation for pressure,

$P_{AA} + \gamma x0.07 = \ \gamma_{Hg}x0.12$

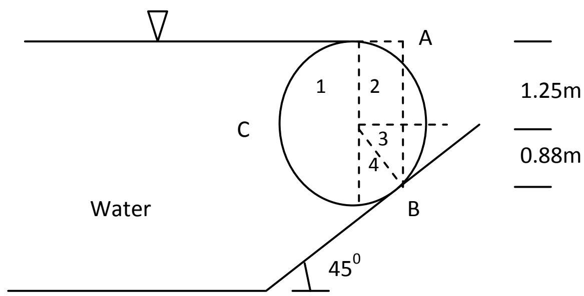
P_{AA} + 0.9x9810x0.07 = 13.6x9810x0.12

P_{AA} = 15392 N/m^2

F_V = P_{AA} A_{bottom}+ Weight of liquid below AA = P_{AA} A_{bottom}+ $\gamma\ Volume_{below\ AA}$

= $P_{AA}A_{bottom}+\gamma\left[Volume_{cylinder\ of\ height\ 16cm}-Volume_{hemisphere\ of\ radius\ 50mm}\right]$
= $15392x\pi x0.05^2+0.9x9810[\pi x0.05^2x0.16-\frac{1}{2}x\frac{4}{3}x\pi x0.05^3]$
= 129.7N (down)

17. The cylinder in the fig. is 1.5m long and its radius is 1.25m. Compute the horizontal and vertical components of the pressure force on the cylinder.



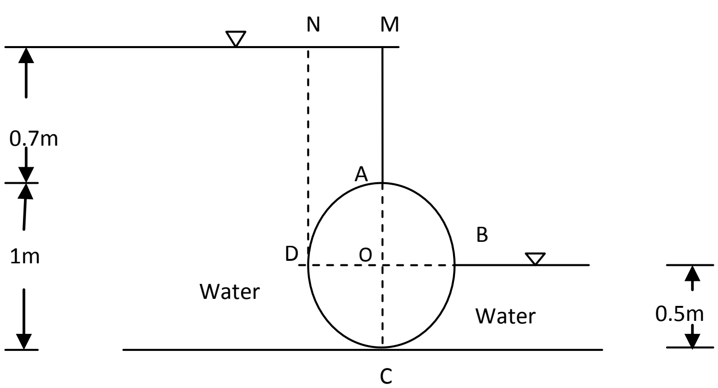
Solution:
AB = 1.25+1.25Sin45 = 1.25+0.88 = 2.13m
Horizontal force $(F_H)=\gamma A\bar{y}$ = 9810x(2.13x1.5)x(2.13/2) = 33380 N = 33.38 KN (right)

Vertical force (Fv) = Weight of volume of water vertically above ABC
= $\gamma\ [Volume\ 1+Volume2+Volume3+Volume4]$

$$=9810x\left[\frac{1}{2}x\pi x1.25^2x1.5+0.88x1.25x1.5+0.5x0.88x0.88x1.5+\frac{1}{8}x\pi x1.25^2x1.5\right]$$

= 67029N = 670.29 KN (up)

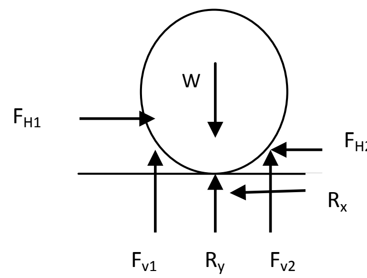
18. The 1m diameter log (sp gr = 0.82) divides two shallow ponds as shown in the fig. Compute the net horizontal and vertical reactions at point C, if the log is 3.7m.



Solution:
Horizontal force on ADC $(F_{H1})=\gamma A1\overline{y1}$ = 9810x3.7x1x1.2= 43556 N (right)

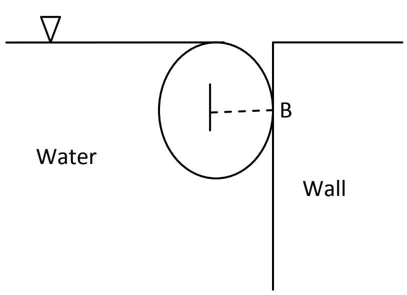
Horizontal force on BC $(F_{H2}) = \gamma A2\overline{y2}$ = 9810x3.7x0.5x0.5/2= 4537 N (left)

Vertical force on ADC (F_{V1}) = Weight of volume of water vertically above ADC
Vertical force (F_{v1}) =$(F_v)_{MNDCOAM}$ (up)- $(F_v)_{MNDAM}$ (down)
$= \gamma Volume_{AOCD}$
$= 9810x\frac{1}{2}x\pi x0.5^2x3.7$ = 14254N (up)
Vertical force on BC (F_{V2}) = Weight of volume of water (imaginary) vertically above BC
$= \gamma\ Volume_{BOC}$
$= 9810x\frac{1}{4}x\pi x0.5^2x3.7$ = 7127N (up)
Weight of log (W) = $\gamma_{log}\ Volume_{log}$
$= 0.82x9810x\pi x0.5^2x3.7$ = 23376N (down)

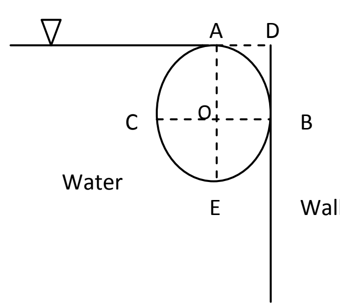


Horizontal reaction at C (R_x)
$-R_X + F_{H1} - F_{H2} = 0$
$R_x = F_{H1} - F_{H2}$ = 43556 – 4537 = 39019N (left)
Vertical reaction at C (R_y)
$R_y + F_{V1} + F_{V2} – W = 0$
R_y = 23376-14254-7127 = 1995N (up)

19. The 0.9m diameter cylinder in the fig. is 7m long and rests in static equilibrium against a frictionless wall at point B. Compute the specific gravity of the cylinder.



Solution:



Vertical force (F_V) = Weight of volume of water vertically above ADBECA

= (F_v) on semi-circle ACE + (F_v) on quadrant BE

For BE, imaginary weight of fluid vertically above it is considered

$$= \gamma\ [Volume_{AOEC} + Volume_{BOE} + Volume_{ADBO}]$$

$$= 9810\left[\frac{1}{2}\pi x 0.45^2 x7 + \frac{1}{4}\pi x 0.45^2 x7 + 0.45x0.45x7\right] = 46670\text{N (up)}$$

The reaction at B is purely horizontal.

Weight of cylinder (W) = F_V

W = 46670N

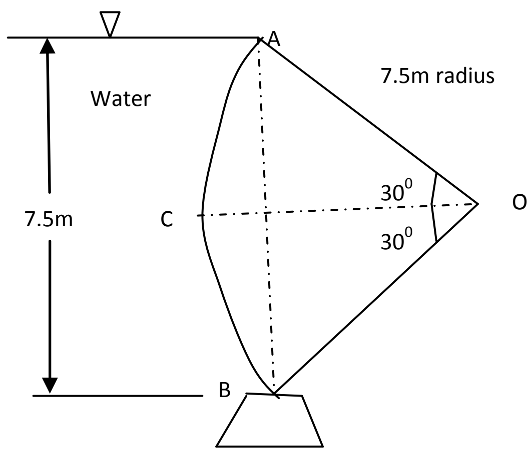
$\gamma_{cyl}\ Volume_{cyl} = 46670$

$\gamma_{cyl}\ \pi x 0.45^2 x 7 = 46670$

γ_{cyl} = 10480 N/m^3

Sp gr of cylinder = $\frac{\gamma_{cyl}}{\gamma} = \frac{10480}{9810}$ = 1.07

20. Find the horizontal and vertical forces per m of width on the tainter gate shown in the fig.



Solution:

Horizontal force $(F_H) = \gamma A\bar{y}$ = 9810x(7.5x1)x7.5/2 = 275906 N = 275.906 KN (right)

F_H acts at a distance of 7.5x2/3 = 5m from water surface.

Vertical force (Fv) = Weight of imaginary volume of water vertically above ABCA

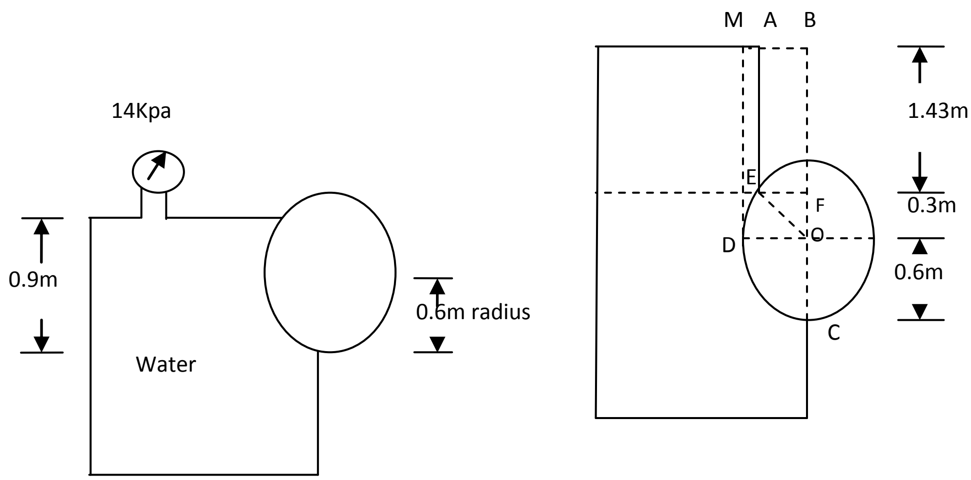
$$= \gamma\ [Volume_{sectorAOBC} - Volume_{triangleAOB}]$$

$$= 9810x\left[\frac{60}{360} x\pi x 7.5^2 x1 - 0.5x7.5x(7.5Cos30)x1\right]$$

= 49986N = 49.986KN (up)

F_V acts through the centroid of the segment ABCA.

21. The tank whose cross section is shown in fig. is 1.2m long and full of water under pressure. Find the components of the force required to keep the cylinder in position, neglecting the weight of the cylinder.



Solution:

Pressure =14KPa

Equivalent head of water = $\frac{P}{\gamma} = \frac{14000}{9810}$ = 1.43m

Apply 1.43m water above the cylinder.

Horizontal force $(F_H) = \gamma A\bar{y}$ = 9810x(0.9x1.2)x(1.43+0.9/2) = 19918 N = 19.918 KN (right)

Sin(OEF) = 0.3/0.6

<OEF = 30^0 = <EOD

EF = 0.6Cos30 = 0.52m

Vertical force (F_v) = $(F_v)_{MABFOCDEM}$ -$(F_v)_{MAED}$

= Weight of volume of water vertically above ABFOCDEA

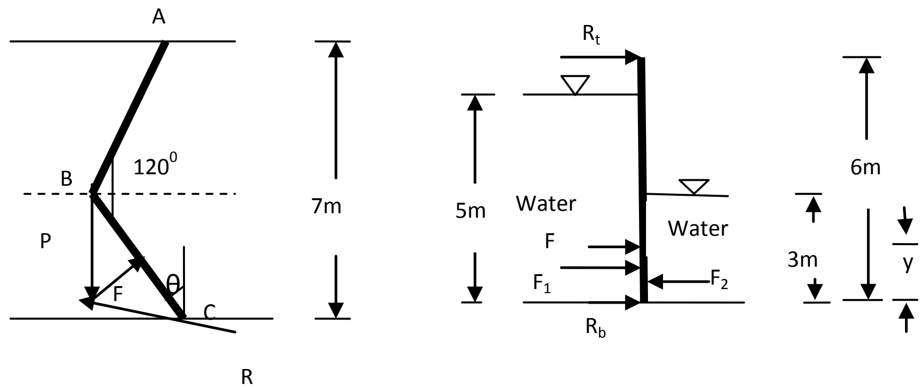
= $\gamma\ [Volume_{\ ABFE} + Volume_{\ triangleEOF} + Volume_{\ sector\ EOD} + Volume_{\ quadrant\ COD}]$

$= 9810x\left[0.52x1.43x1.2 + 0.5x0.3x0.52x1.2 + \frac{30}{360}x\pi x0.6^2x1.2 + \frac{1}{4}x\pi x0.6^2x1.2\right]$

= 14110 N = 14.11 KN (up)

Forces required to keep the cylinder in positions are: 19.918KN to the right and 14.11KN to the up.

22. Each gate of a lock is 6m high and is supported by two hinges placed on the top and the bottom. When the gates are closed, they make an angle of 120^0. The width of the lock is 7m. If the water levels are 5m and 2m at upstream and downstream respectively, determine the magnitude of forces on the hinge due to the water pressure.



Solution:

F = Resultant water force, P = Reaction between gates, R = total reaction at hinge

$\theta = 30^0$

Width of lock = 3.5/cos30 = 4.04m

Resolving forces along gate

$P\cos\theta = R\cos\theta$ i.e. P=R (a)

Resolving forces normal to gate

$P\sin\theta + R\sin\theta = F$ (b)

From a and b

$P = F/2\sin\theta$

Horizontal force on upstream side $(F_1) = \gamma A1\overline{y1}$ = 9810x4.04x5x5/2= 495405 N

F1 acts at 5/3m = 1.66 from bottom

Horizontal force on downstream side $(F_2) = \gamma A2\overline{y2}$ = 9810x4.04x3x3/2= 178346N

F2 acts at 3/3 = 1m from bottom

$F = F_1 - F_2$ = 495405-178346 = 317059 N

Taking moment about the bottom to find the point of application of F,

317059y = 495405x1.66-178346x1

y = 2.03m

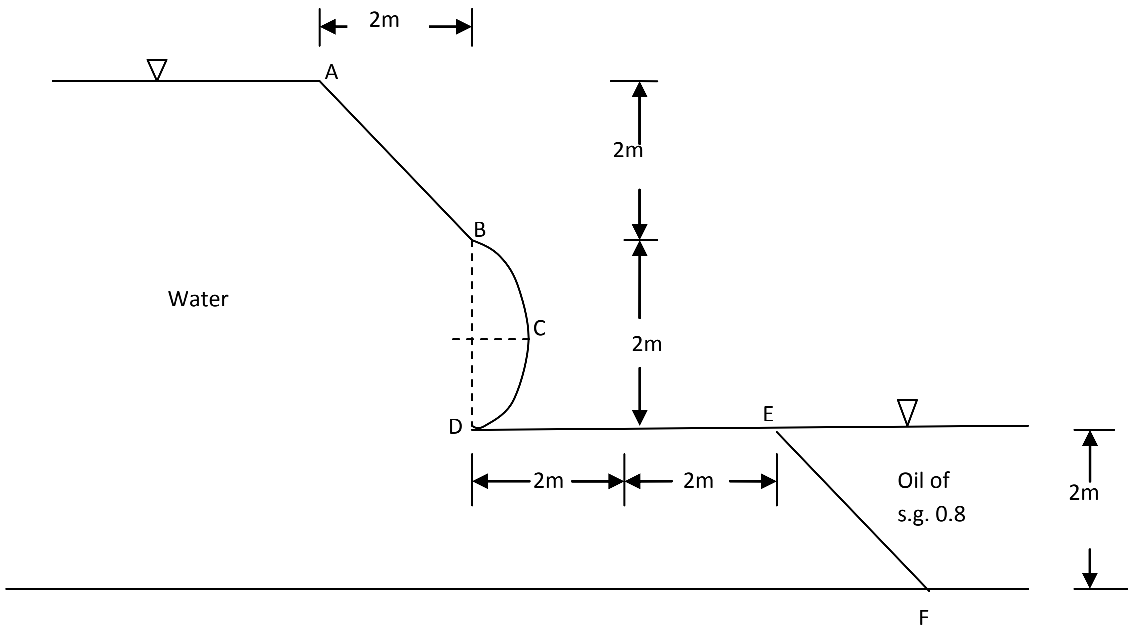
$P = F/2\sin\theta$ = 317059/2sin30 =317059N

R = P = 317059N

Taking moment about bottom hinge

R_t x6 =317059x2.03
R_t = 107272N = 107.272KN
$R_b = R - R_t$ = 317059-107272 = 209787N = 209.787KN

23. Find the net horizontal and vertical forces acting on the surface ABCDEF of width 5m as shown in the figure below. BCD is a half circle.



Solution:
AB = 2/Sin45^0 = 2.8284m
EF = 2/Sin45^0 = 2.8284m

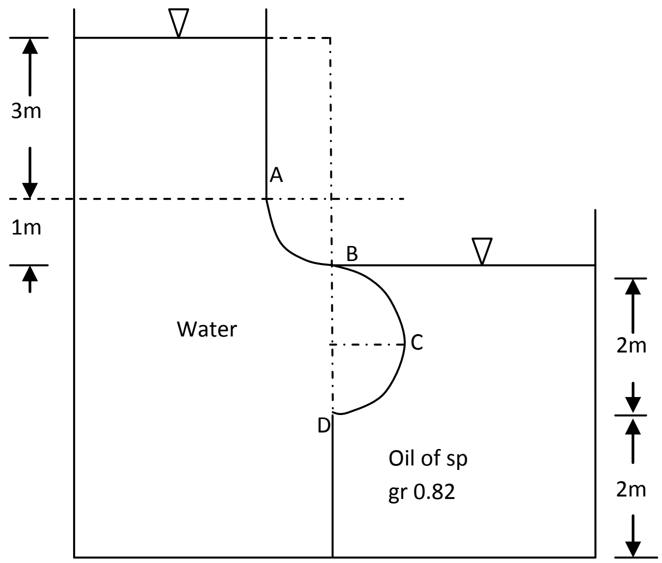
Pressure force on inclined surface AB $(F_1) = \gamma_{water} A_1 \overline{y_1}$ = 9810x(2.8284x5)x1 = 138733N which is perpendicular to AB
$F_{1x} = F_1$ Cos45^0 = 138733 Cos45^0 = 98099N (right)
$F_{1y} = F_1$ Sin45^0 = 138733 Sin45^0 = 98099N (up)
For curved surface BCD
$F_{2x} = \gamma_{water} A_2 \overline{y_2}$ = 9810x(2x5)x3 = 294300 N (right)
$F_{2y} = \gamma_{water} V_{above\ BCD} = 9810x\left(\frac{1}{2}\pi x \frac{2^2}{4}\right)x5$ = 77048N (down)

Pressure force on EF due to water $(F_3) = \gamma_{water} A_3 \overline{y_3}$ = 9810x(2.8284x5)x5 = 693665N which is perpendicular to EF
$F_{3x} = F_3$ Cos45^0 = 693665 Cos45^0 = 490495N (right)
$F_{3y} = F_3$ Sin45^0 = 693665 Sin45^0 = 490495N (up)

Pressure force on EF due to oil $(F_4) = \gamma_{oil} A_4 \overline{y_4}$ = 0.8x9810x(2.8284x5)x1 = 110986N which is perpendicular to EF

F_{4x} = F_4 Cos45^0 = 1109955 Cos45^0 = 78479N (left)
F_{4y} = F_4 Sin45^0 1109955 Sin45^0 =78479N (down)
Net horizontal force = 98099+294300+490495-78479N = 804415N (right)
Net vertical force = 98099-77048+490495-78479N = 433067N (up)

24. Calculate the pressure force on the curved surface ABCD as shown in the figure below. AB is a quadrant of radius 1m and BCD is a semi-circle of radius 1m. Take width of curve = 5m.



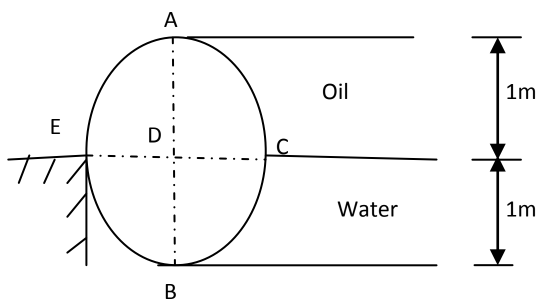
Solution:
Horizontal force on AB $(F_{1x}) = \gamma_{water} A_1 \overline{y_1}$ = 9810x(1x5)x3.5 = 171675N (right)
Vertical force on AB $(F_{1y)} = \gamma_{water} V_{above\ AB\ (imaginary)} = 9810x\left(\frac{1}{4}\pi x 1^2 + 3x1\right)x5$ = 185674N (up)

Horizontal force on BCD from the left side $(F_{2x}) = \gamma_{water} A_2 \overline{y_2}$ = 9810x(2x5)x5 = 490500N (right)
Vertical force on BCD from the left side $(F_{2y)} = \gamma_{water} V_{above\ BCD} = 9810x\left(\frac{1}{2}\pi x 1^2\right)x5$ = 77048N (down)

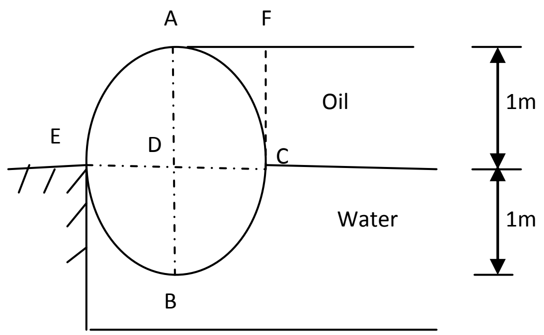
Horizontal force on BCD from the right side $(F_{3x}) = \gamma_{oil} A_2 \overline{y_3}$ = 0.82x9810x(2x5)x1 = 80442N (left)
Vertical force on BCD from the right side $(F_{3y)} = \gamma_{oil} V_{above\ BCD} = 0.82x9810x\left(\frac{1}{2}\pi x 1^2\right)x5$ = 63179N (up)

Net horizontal force = 171675+490500-80442 = 581733N = 581.733KN (right)
Net vertical force = 185674-77048+63179 = 171805N = 171.805KN (up)

25. Find the weight of the cylinder (dia. =2m) per m length if it supports water and oil (sp gr = 0.82) as shown in the figure. Assume contact with wall as frictionless.



Solution:



Downward force on AC due to oil (FV_{AC}) = Weight of oil supported above curve AC

$= \gamma_{oil}\ Volume\ of\ oil\ above\ AC$

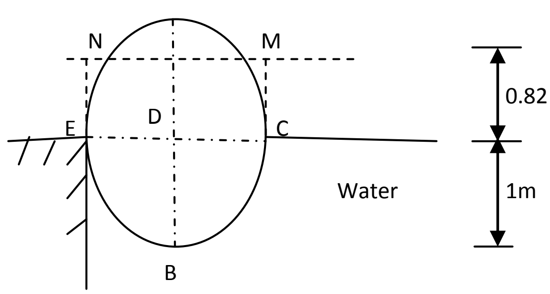
$= \gamma_{oil}\ (Volume_{AFCD} - Volume_{\ quadrant\ ACD})$

$= 0.82x9810\left(1x1x1 - \frac{1}{4}\pi x1^2x1\right)$ = 1726N

Pressure at C due to 1m oil (P) = $\gamma_{oil}\ x1$ = 0.82x9810x1 = 8044.2 Pa

Equivalent head of water due to 1m oil = $\frac{P}{\gamma} = \frac{8044.2}{9810}$ = 0.82m

Apply 0.82m water above EC.



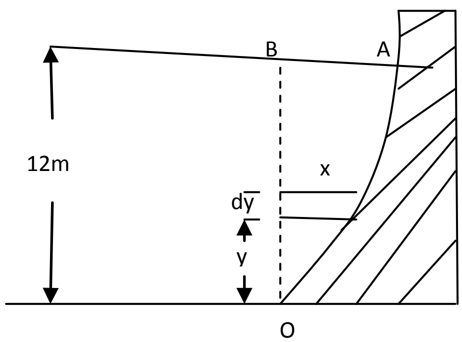
Upward vertical force on CBE (FV_{CBE}) = Weight of water above CBE

$= \gamma(Volume_{\ semi\ circle\ CBE} + Volume_{\ CMNE})$

$$= 9810\left(\frac{1}{2}\pi x1^2x1 + 0.82x2x1\right) = 31498N$$

Weight of cylinder = FV_{CBE} - FV_{AC} = 31498-1726 = 29772 N

26. Find the magnitude and direction of the resultant pressure force on a curved face of a dam which is shaped according to the relation y = $x^2/6$. The height of water retained by the dam is 12m. Assume unit width of the dam.



Solution:

The equation of the dam

y = $x^2/6$

$x = \sqrt{6y}$

Consider an element of thickness dy and length x at a distance y from the base.

Area of element = xdy

Area of OAB = $\int_0^{12} xdy = \int_0^{12} \sqrt{6y}dy$

$= \sqrt{6}x\frac{2}{3}\left|y^{3/2}\right|_0^{12}$ = 67.882 m^2

Horizontal force $(F_x) = \gamma A\bar{y}$ = 9810x(12X1)x6 = 706320N

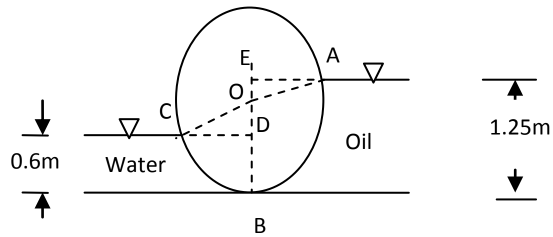
Vertical force (F_y) = Weight of water vertically above dam OA

= $\gamma\ Vol_{OAB} = \gamma\ Area_{OAB}L$ = 9810x67.882x1 = 665922N

Resultant force $(F_R) = \sqrt{F_{x+}^2 F_y^2}$ = 970742N = 970.742KN

Direction of resultant force = $Tan^{-1}\frac{F_y}{F_x}$ = $Tan^{-1}\frac{665922}{706320}$ = 43.31^0

27. A cylinder, 2m in diameter and 3m long weighing 3KN rests on the floor of the tank. It has water to a depth of 0.6m on one side and liquid of sp gr 0.7 to a depth of 1.25m on the other side. Determine the magnitude and direction of the horizontal and vertical components of the force required to hold the cylinder in position.



Solution:

OA= OB = OC = 1m, BD = 0.6m

OD = 1-0.6 = 0.4m

CD = $(1^2 - 0.4^2)^{1/2}$= 0.9165m

$< COD = tan^{-1}\left(\frac{0.9165}{0.4}\right)$= 66.4^0

OE = 1.25-1 = 0.25m

$< AOE = cos^{-1}\left(\frac{0.25}{1}\right)$= 75.5^0

<AOB = 180-75.5 = 104.5^0

AE = 0.25 tan75.5 = 0.96m

Weight of cylinder = 3KN = 3000N

Net horizontal force $(F_H) = (F_H)_{AB} - (F_H)_{CB} = \gamma_{oil} A1\overline{y1} - \gamma A2\overline{y2}$

= 0.7x9810x1.25x3x1.25/2-9810x0.6x3x0.6/2 = 10797N (left)

Net vertical force (F_V)= Weight of volume of oil vertically above AB + Weight of volume of water vertically above BC = Fv_{AB} (up) + Fv_{BC} (up)

= $\gamma_{oil}\ Volume_{AEB} + \gamma Volume_{BDC}$

$= \gamma_{oil}(Volume\ of\ sector\ AOB + Volume\ of\ \Delta AOE) +$

$\gamma(Volume\ of\ sector\ BOC\ - Volume\ of\ \Delta COD)$

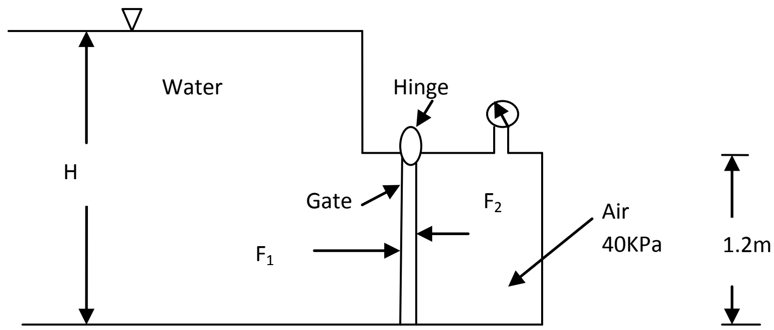
=$9810x0.7\left(\frac{104.5}{360}x\pi x1^2x3 + 0.5x0.25x0.96x3\right) + [9810x\ \left(\frac{66.4}{360}x\pi x1^2x3 - 0.5x0.9165x0.4x3\right)]$

= 32917N (up)

The components to hold the cylinder in place are 10797 N to the right and 32917-3000 = 29917N down.

Additional problems on hydrostatic force

For the system shown in figure, calculate the height H of water at which the rectangular hinged gate will just begin to rotate anticlockwise. The width of gate is 0.5m.



Solution:

Force due to water $(F_1) = \gamma A\bar{y}$

$$= 9810x(1.2x0.5)x(H - 0.6) = 5886(H - 0.6)$$

CP of F_1

M.I. about CG $(I_G) = \frac{1}{12}x0.5x1.2^3$ = 0.072m^4

Vertical distance of CP of F_1 from free surface

$y_{p1} = \bar{y} + \frac{I_G}{A\bar{y}} = (H - 0.6) + \frac{0.072}{1.2x0.5(H-0.6)} = \frac{(H-0.6)^2+0.12}{(H-0.6)}$

Force due to air pressure (F_2)= PA = 40x1000x1.2x0.5 =24000N, which acts at a distance of H-0.6 from the free surface.

Taking moment about hinge,

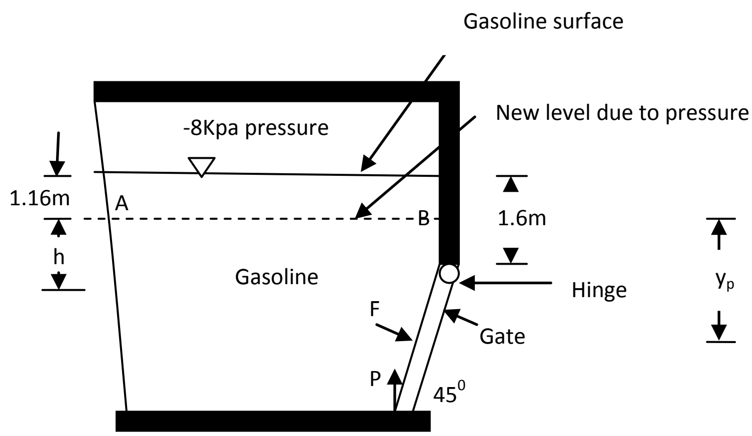
$F_1\left[y_{p1} - (H - 1.2)\right] = F_2x0.6$

$5886(H - 0.6)\left[\frac{(H-0.6)^2+0.12}{(H-0.6)} - (H - 1.2)\right] = 24000x0.6$

$(H - 0.6)^2 + 0.12 - (H - 1.2)(H - 0.6) = 2.446$

H = 1.6m

A 3m square gate provided in an oil tank is hinged at its top edge. The tank contains gasoline (sp. gr. = 0.7) up to a height of 1.6m above the top edge of the plate. The space between the oil is subjected to a negative pressure of 8 Kpa. Determine the necessary vertical pull to be applied at the lower edge to open the gate.



Solution:

Head of oil equivalent to -8 Kpa pressure = $\frac{p}{\gamma_{oil}} = \frac{-8000}{0.7x9810}$ = -1.16m

This negative pressure will reduce the oil surface by 1.16m. Let AB = new level. Make calculation by taking AB as free surface.

h = 1.6-1.16= 0.44m

$\bar{y} = (1.6 - 1.16) + \frac{1}{2}x3Sin45$ = 1.5m

Hydrostatic force $(F) = \gamma_{oil}A\bar{y}$ = 0.7x9810x(3x3)x1.5 = 92704.5N

CP of F

M.I. about CG $(I_G) = \frac{1}{12}x3x3^3$ = 6.75m^4

Vertical distance of CP of F_1 from free surface

$y_p = \bar{y} + \frac{I_G Sin^2\theta}{A\bar{y}} = 1.5 + \frac{6.75Sin^2 45}{(3x3)x1.5}$ = 1.75m

Vertical distance between the hinge and F = 1.75-0.44 = 1.31m

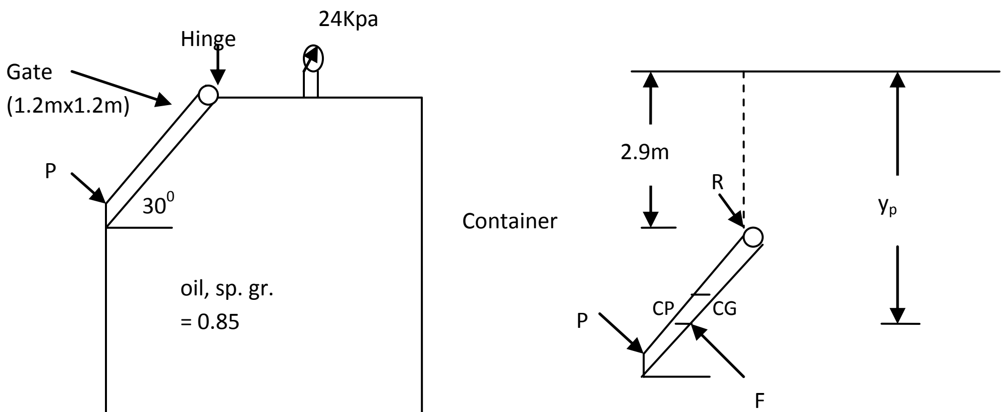
Taking moment about the hinge

$Px3sin45 = Fx\frac{1.31}{sin45}$

$Px3sin45 = 92704.5x\frac{1.31}{sin45}$

P = 80962N

There is an opening in a container shown in the figure. Find the force P and the reaction at the hinge (R).



Solution:

Equivalent head of oil due to 24 Kpa pressure = $\frac{p}{\gamma_{oil}} = \frac{24000}{0.85x9810}$ = 2.9m of oil

Apply 2.9m of oil above the hinge.

$\bar{y} = 2.9 + \frac{1}{2}x1.2Sin30$ = 3.2m

Hydrostatic force $(F) = \gamma_{oil}A\bar{y}$ = 0.85x9810x(1.2x1.2)x3.2= 38424N

CP of F

M.I. about CG $(I_G) = \frac{1}{12}x1.2x1.2^3$ 0.1728m^4

Vertical distance of CP of F_1 from free surface

$y_p = \bar{y} + \frac{I_G Sin^2\theta}{A\bar{y}} = 3.2 + \frac{0.1728Sin^2 30}{(1.2x1.2)x3.2}$ = 3.209m

Vertical distance between the hinge and F = 3.209-2.9 = 0.309m

Taking moment about the hinge

$Px1.2 = 38424x\frac{0.309}{sin30}$

P = 19788N

R+P=F

R = 38424-19788 = 18636N