Tutorial 4

Buoyancy and floatation

1. A rectangular pontoon has a width of 6m, length of 10m and a draught of 2m in fresh water. Calculate (a) weight of pontoon, (b) its draught in seawater of density 1025 kg/m^3 and (c) the load that can be supported by the pontoon in fresh water if the maximum draught permissible is 2.3m.

Solution:

a) Weight of pontoon = Weight of water displaced

W = $\gamma_{water} V_{water\ displaced}$ = 9810x6x10x2 = 1177200 N = 1177.2 KN

b) Draught in sea water (D) = ?

Weight of pontoon = Weight of sea water displaced

1177200 = $\gamma_{sea\ water} V$

1177200 = $1025x9.81x6x10xD$

D = 1.95m

c) D_{max} = 2.3m

Load that can be supported in fresh water (P) = ?

Total upthrust (F_B) = Weight of water displaced

= $\gamma_{water} V_{water\ displaced}$ = 9810x6x10x2.3 = 1353780N = 1353.78KN

P = F_B – W = 1353.78-1177.2= 176.58 KN

2. A steel pipeline carrying gas has an internal diameter of 120cm and an external diameter of 125cm. It is laid across the bed of a river, completely immersed in water and is anchored at intervals of 3m along its length. Calculate the buoyancy force per meter run and upward force on each anchorage. Take density of steel = 7900 kg/m^3.

Solution:

Buoyant force per m = Weight of water displaced per m

= $\gamma_{water} V = 9810x\frac{\pi}{4}x1.25^2x1$ = 12039N/m

Buoyant force for 3m (F_{B3}) = 12039x3 = 36117N

Weight for 3 m of pipe (W_3) = $3x\gamma_{steel} V_{steel}$

$= 3x7900x9.81x\frac{\pi}{4}x(1.25^2 - 1.20^2)$ = 22369N

Upward force on each anchorage = F_{B3} – W_3 = 36117-22369 = 13748N

3. A wooden block of width 2m, depth 1.5m and length 4m floats horizontally in water. Find the volume of water displaced and the position of center of buoyancy. The specific gravity of wooden block is 0.7.

Solution:

Weight of block = Weight of water displaced

$\gamma_{wood}V_{wood} = \gamma_{water}V_{water\ diplaced}$

0.7x9810x2x1.5x4 = 9810$V_{water\ displaced}$

$V_{water\ displaced}$ = 8.4m^3

Finding depth of immersion (h)

Weight of block = Weight of water displaced

$\gamma_{wood}V_{wood} = \gamma_{water}V_{water\ diplaced}$

0.7x9810x2x1.5x4 = 9810x2x4xh

h = 1.05m

Position of center of buoyancy = h/2 = 0.525m from bottom

4. A piece of wood of sp gr 0.65 is 80mm square and 1.5m long. How many Newtons of lead weighing 120KN/m^3 must be fastened at one end of the stick so that it will float upright with 0.3m out of water?

Solution:

Length of wood = 1.5m

Length of wood in water = 1.5-0.3 = 1.2m

Total weight of wood and lead = Weight of water displaced

$\gamma_{wood}V_{wood} + \gamma_{lead}V_{lead} = \gamma_{water}V_{water\ diplaced}$

$0.65x9810x0.08^2x1.5 + \gamma_{lead}V_{lead} = 9810[0.08^2x1.2 + V_{lead}]$

$61.214 + 120000V_{lead} = 75.34 + 9810V_{lead}$

V_{lead} = 0.000128 m^3

Weight of lead = $\gamma_{lead}V_{lead}$ = 120000x0.000128 = 15.38N

5. A block of wood floats in water with 40mm projecting above the water surface. When placed in glycerin of sp gr 1.35, the block projects 70mm above the surface of that liquid. Determine the sp gr of wood.

Solution:

A = Area of block

h = Height of block

S = sp gr of block

Weight of wooden block (W) = $\gamma_{wood}V_{wood} = Sx9810xAh$ (a)

Weight of water displaced (W_W) $= \gamma_{water}V_{water\ diplaced}$ = 9810xA(h-0.04) (b)

Weight of glycerin displaced (W_G) $= \gamma_{glycerin}V_{glycerin\ diplaced}$ = 1.35x9810(h-0.07) (c)

Here, W = W_W = W_G

Equating a and b

Sx9810xAh= 9810xA(h-0.04)

$S = \frac{h-0.04}{h}$

Equating b and c

9810A(h-0.04) = 1.35x9810A (h-0.07)

h = 0.155m

$S = \frac{h-0.04}{h} == \frac{0.155-0.04}{0.155}$ =0.74

6. A rectangular open box, 7.6m by 3m in plan and 3.7m deep, weighs 350KN and is launched in fresh water. (a) How deep will it sink? (b)If the water is 3.7m deep, what weight of stone placed in the box will cause it to rest on the bottom?

Solution:

a) Depth of immersion (h) = ?

Weight of block = Weight of water displaced

$350000 = \gamma_{water} V_{water\ diplaced}$

350000 = 9810x7.6x3xh

h = 1.56m

b) Weight of block and stone = Weight of water displaced

$350000 + Weight_{stone} = \gamma_{water} V_{water\ diplaced}$

$350000 + Weight_{stone} = 9810x\ 7.6x3x3.7$

$Weight_{stone}$ = 477572N = 477.57KN

7. A stone weighs 500 N in air and 200N in water. Determine the volume of stone and its specific gravity.

Solution:

Weight in air-Weight in water = Buoyant force

Buoyant force (F_B) =500-200 = 300N

F_B = Weight of water displaced

$300 = \gamma_{water} V_{water\ diplaced}$

$300 = 9810x V_{water\ diplaced}$

$V_{water\ diplaced}$ = 0.0306m^3

Volume of stone (V) = 0.306m^3

Specific weight of stone $(\gamma_{stone}) = \frac{Weight\ in\ air}{V} = \frac{500}{0.0306}$ = 16340N/m^3

Specific gravity of stone $= \frac{\gamma_{stone}}{\gamma_{water}} = \frac{16340}{9810}$ = 1.66

8. A metallic body floats at the interface of mercury of specific gravity 13.6 and water in such a way that 30% of its volume is submerged in mercury and 70% in water. Find the density of the metallic body.

Solution:

Volume of metallic body = V

Density of metallic body = ρ

Weight of metallic body = Weight of fluid displaced = Buoyant force due to mercury + Buoyant force due to water

$\rho g V = \rho_{mercury} g V_{mercury\ diplaced} + \rho_{water} g V_{water\ diplaced}$

$\rho x 9.81 x V = 13.6x1000x9.81x0.3V + 1000x9.81x0.7V$

ρ = 4780 kg/m^3

9. A wooden block 4mx1mx0.5m is floating in water. Its specific gravity is 0.76. Find the volume of the concrete of specific gravity 2.5, that may be placed on the block which will immerse the (a) block completely in water and (b) block and concrete completely in water.

Solution:

a) Immersion of the block completely in water

Total weight of block and concrete = Weight of water displaced

$\gamma_{wood} V_{wood} + \gamma_{concrete} V_{concrete} = \gamma_{water} V_{water\ diplaced}$

$0.76x9810x4x1x0.5 + 2.5x9810xV_{concrete} = 9810x4x1x0.5$

$V_{concrete}$ = 0.192m^3

b) Immersion of the block and concrete completely in water

Total weight of block and concrete = Weight of water displaced

$\gamma_{wood} V_{wood} + \gamma_{concrete} V_{concrete} = \gamma_{water} V_{water\ diplaced}$

$0.76x9810x4x1x0.5 + 2.5x9810xV_{concrete} = 9810x[4x1x0.5 + V_{concrete}]$

$V_{concrete}$ = 0.32m^3

10. Determine the specific weight and volume of an object that weighs 10N in water and 12N in oil of specific gravity 0.8.

Solution:

Weight of object = W

Volume of object = V

Weight of object – Weight in water = Buoyant force due to water

$W - 10 = \gamma_{water} V_{water\ diplaced}$

$W - 10 = 9810V$

$W - 10 = 9810V$ (a)

Weight of object – Weight in oil = Buoyant force due to oil

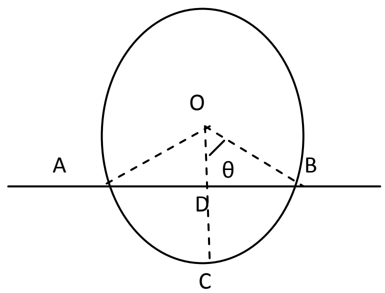
$W - 10 = \gamma_{oil} V_{oil\ diplaced}$

$W - 12 = 0.8x9810xV$

$W - 12 = 7848V$ (b)
Solving a and b,
V = 0.001019 m^3
W = 10 + 9810x0.001019 = 19.996N

Specific weight of body $(\gamma_b) = \frac{W}{V} = \frac{19.996}{0.001019}$ = 19623 N/m^3

11. To what depth will a 2.4m diameter log 5m long and sp gr 0.4 sink in fresh water?



Solution:
Radius (r) = 1.2m
Depth of floatation = DC = ?

DB = 1.2Cosθ, DO =1.2Sinθ

Weight of log = Weight of water displaced
$\gamma_{log}V_{log} = \gamma_{water}V_{water\ diplaced}$
$0.4x9810x\pi x1.2^2x5 = 9810[V_{sector\ OABC} - V_{2\ traingles}]$
$9810x9.04 = 9810\left[\frac{2\theta}{360}x\pi x1.2^2x5 - 2x0.5x1.2cos\theta x1.2sin\theta\right]$
$9.04 = 0.1256\theta - 0.72sin2\theta$
Solving by trial and errors for θ
$\theta = 80$, Right side = 9.8
$\theta = 75$, Right side = 9.06
$\theta = 74.9$, Right side = 9.04

Take $\theta = 74.9$
Depth of floatation (DC) = OC – OD = 1.2 – 1.2Sin74.9 = 0.041m

12. What fraction of the volume of solid piece of metal of sp gr 7.2 floats above the surface of a container of mercury of sp. gr. 13.6?

Solution:
V = Volume of metal

V' = Volume of mercury displaced

Weight of body = Weight of mercury displaced

$\gamma_{body} V_{body} = \gamma_{mercury} V_{mercury\ diplaced}$

7.2x9810xV = 13.6x9810xV'

V'/V = 0.53

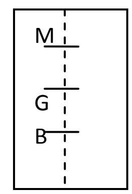
Fraction of volume above mercury = 1-0.53 = 0.47

13. A uniform body of size 4mx2mx1m floats in water. What is the weight of the body if the depth of immersion is 0.6m? Also determine the meta-centric height.

Solution:

Weight of body = Weight of water displaced

$= \gamma_{water} V_{water\ diplaced}$ = 9810x4x2x0.6 = 47088 N



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Position of center of buoyancy (OB) = 0.6/2 = 0.3m

Position of cg (OG) = 1/2 = 0.5m

$MB = \frac{I}{V} = \frac{\frac{1}{12} x 4 x 2^3}{4x2x0.6}$ = 0.556m

BG = 0.5-0.3 = 0.2m

GM = MB-BG = 0.556-0.2 = 0.356m

14. A solid cylinder of diameter 3m has a height of 2m. Find the meta-centric height of cylinder when it is floating in water with its axis vertical. The specific gravity of cylinder is 0.7.

Solution:

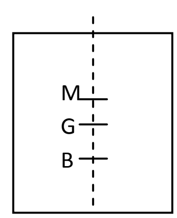
h = depth of immersion

Weight of body = Weight of water displaced

$\gamma_{cyl} V_{cyl} = \gamma_{water} V_{water\ diplaced}$

$0.7x9810x\pi x1.5^2x2 = 9810x\pi x1.5^2xh$

h = 1.4m



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Position of center of buoyancy (OB) = 1.4/2 = 0.7m

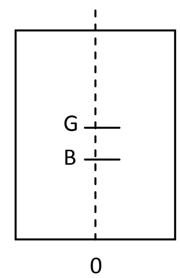
Position of cg (OG) = 2/2 = 1m

BG = 1-0.7 = 0.3m

$MB = \frac{I}{V} = \frac{\frac{1}{4} x\pi x 1.5^4}{\pi x 1.5^2 x 1.4}$ = 0.4017m

GM = MB-BG = 0.4017-0.3 = 0.1017m

15. A solid wood cylinder has a diameter of 0.6m and a height of 1.2m. The sp.gr. of the wood is 0.6. If the cylinder is placed vertically in oil of sp.gr. 0.85, would it be stable?



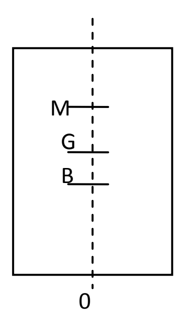
Solution:
h = depth of immersion
Weight of body = Weight of oil displaced
$\gamma_{cyl}V_{cyl} = \gamma_{oil}V_{oil\ diplaced}$
$0.6x9810x\pi x0.3^2x1.2 = 0.85x9810x\pi x0.3^2xh$
h = 0.847m

Position of center of buoyancy (OB) = 0.847/2 = 0.4235
Position of cg (OG) = 1.2/2 = 0.6m
BG = 0.6-0.4235 = 0.1765m

$MB = \frac{I}{V} = \frac{\frac{1}{4}x\pi x0.3^4}{\pi x0.3^2x0.847}$ = 0.0265m

GM = MB-BG = 0.0265-0.1765 = -0.15m
As metacentric height GM is negative, the body is unstable.

16. A body of size 3mx2mx2m floats in water. Find the limit of weight of the body for stable equilibrium.



Solution:
S = Sp gr of plastic
h = depth of immersion
Weight of body = Weight of water displaced
$\gamma_{body}V_{body} = \gamma_{water}V_{water\ diplaced}$
$Sx9810x3x2x2 = 9810x3x2xh$
h =2S

Position of center of buoyancy (OB) = 2S/2 = S
Position of cg (OG) = 2/2 = 1m
BG = OG-OB =1-S

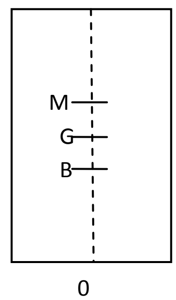
$MB = \frac{I}{V} = \frac{\frac{1}{12}x3x2^3}{3x2x2S} = \frac{0.166}{S}$

GM = MB-BG = $\frac{0.166}{S} - 1 + S$
For stable equilibrium, GM > 0

$\frac{0.166}{S} - 1 + S > 0$
S^2-S+0.166>0
Values of S are 0.21 and 0.79
Lower limit of Weight = $\rho_{body} g V_{body}$ = 0.21x1000x9.81x3x2x2 = 24721N = 24.72KN
Upper limit of weight = $\rho_{body} g V_{body}$ = 0.79x1000x9.81x3x2x2 = 92999N = 92.99KN

17. A wooden cylinder of specific gravity 0.6 and circular in cross-section is required to float in oil of specific gravity 0.8. Calculate the ratio of length to diameter for the cylinder so that it will just float upright in water.

Solution:
Length of cylinder = L
Diameter of cylinder = D



h = depth of immersion
Weight of body = Weight of oil displaced
$\gamma_{cyl} V_{cyl} = \gamma_{oil} V_{oil\ diplaced}$
$0.6x9810x\frac{\pi}{4}xD^2xL = 0.8x9810x\frac{\pi}{4}xD^2xh$
h = 0.75L

Position of center of buoyancy (OB) = 0.75L/2 = 0.375L
Position of cg (OG) = L/2 = 0.5L
BG = OG-OB =0.5L-0.375L = 0.125L

$$MB = \frac{I}{V} = \frac{\frac{1}{64}x\pi xD^4}{\frac{\pi}{4}xD^2x0.75L} = \frac{D^2}{12L}$$

GM = MB-BG = $\frac{D^2}{12L} - 0.125L$
For stable equilibrium, GM > 0
$\frac{D^2}{12L} - 0.125L > 0$

D^2-1.5L^2>0
L^2/D^2 <0.667
L/D<0.8167

18. A solid cone of specific gravity 0.7 floats in water with its apex downwards. Determine the least apex angle of the cone for equilibrium.

Solution:

D = Dia. Of cone

d =Dia. Of cone at water surface

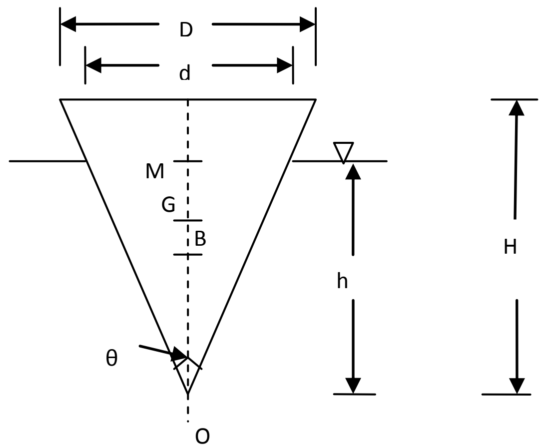
h = Depth of immersion

H = Height of cone

2θ = Apex angle

R = Radius of cone

r = radius of cone at water surface



Finding depth of immersion (h)

Weight of cone = Weight of water displaced

$\gamma_{cone} V_{cone} = \gamma_{water} V_{water\ diplaced}$

$0.7x9810x\frac{1}{3}x\pi x R^2 x H = 9810x\frac{1}{3}x\pi x r^2 x h$

$h = \frac{0.7R^2H}{r^2}$

$h = \frac{0.7(HTan\theta)^2 H}{(hTan\theta)^2}$

h = 0.8879H

OG = $\frac{3}{4}H$ = 0.75H

OB = $\frac{3}{4}h$ = 0.75x0.8879H = 0.6659H

BG = OG-OB = 0.75H-0.6659H = 0.0841H

$MB = \frac{I}{V} = \frac{\frac{1}{4}x\pi x r^4}{\frac{1}{3}x\pi x r^2 x h} = \frac{0.75r^2}{h}$

$= \frac{0.75(hTan\theta)^2}{h} = 0.75hTan^2\theta = 0.75x0.8879HTan^2\theta = 0.6659HTan^2\theta$

GM = MB-BG = $0.6659HTan^2\theta - 0.0841H$

For stable equilibrium GM>0

$0.6659HTan^2\theta - 0.0841H > 0$

$Tan^2\theta - 0.1263 > 0$

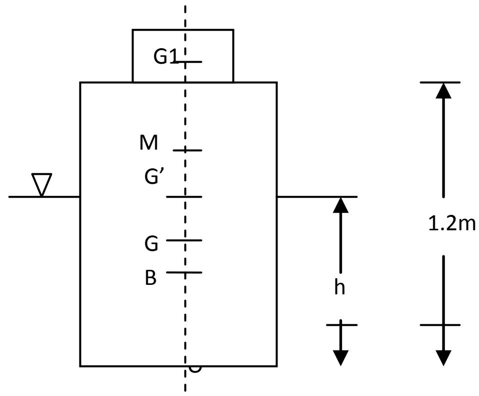
$Tan\theta > 0.3553$

$\theta > 19.56^0$

Least apex angle = 2θ = 2x19.56 = 39.12^0

19. A cylindrical buoy 1.8m in diameter, 1.2m high and weighing 10.5 KN floats in salt water of density 1025 kg/m^3. Its CG is 0.45m from the bottom. If a load of 3KN is placed on the top, find the maximum height of the CG of this load above the bottom if the buoy is to remain in stable equilibrium.

Solution:



G = CG of buoy

G1 = CG of load 3KN

G' = Combined CG of load and buoy

h = depth of immersion

B = CB

OG =0.45m

OG1=?

Finding depth of immersion (h)

Weight of load and buoy = Weight of water displaced

$10500 + 3000 = \rho_{salt\ water} g V_{water\ diplaced}$

$13500 = 1025x9.81x\pi x0.9^2xh$

h = 0.53m

OB = 0.53/2 = 0.265m

$MB = \frac{I}{V} = \frac{\frac{1}{64}x\pi x1.8^4}{\frac{\pi}{4}x1.8^2x0.53}$ = 0.38m

BG' = OG'-OB = OG'-0.265

G'M = MB-BG' = 0.38- OG'+0.265 = 0.645-OG'

For stable equilibrium, G'M>0

0.645-OG'>0

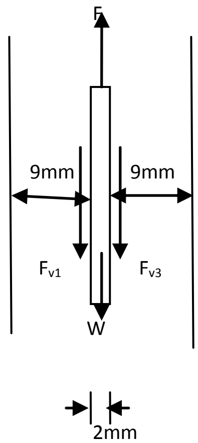
OG' < 0.645m

Taking moments about O,

$3x10^3xOG1 + 10.5x10^3x0.45 = (3x10^3+10.5x10^3)x0.645$

OG1 = 1.3275m

20. A plate of metal 1.1mx1.1mx2mm is to be lifted up with a velocity of 0.1m/s through an infinitely extending gap 20mm wide containing an oil of sp. gr. 0.9 and viscosity 2.1NS/m^2. Find the force required to lift the plate assuming the plate to remain midway in the gap. Assume the weight of the plate to be 30N.



Solution:

Velocity of plate (u) = 0.1m/s

Sp. gr. of oil = 0.9

Sp. wt. of oil (γ) = 0.9x9810 = 8829N/m^3

Viscosity of oil (μ) = 2.1NS/m^2

Clearance on both sides ($dy_1 = dy_2 = dy$) = 9mm = 0.009m

Weight of plate = 30N

Force required to lift the plat (F) = ?

Upthrust on the plate = γxvol. of plate = 8829x1.1x1.1x2/1000 = 21.37N

Viscous force on the plate = Viscous force on left + Viscous force on right

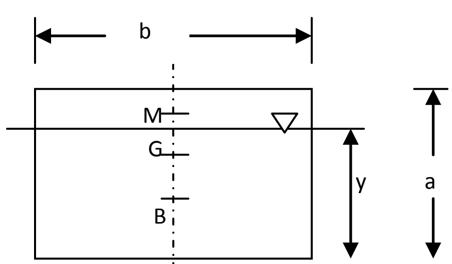
= $\tau_1 A + \tau_2 A = (\tau_1 + \tau_2) A$

$= \left(\mu \frac{du}{dy} + \mu \frac{du}{dy}\right) A = 2\mu \frac{du}{dy} A = 2x2.1x\frac{0.1}{0.009}x1.1x1.1$ = 56.47N

Total force required to lift the plate = Weight-Upthrust + Viscous force = 30-21.37+56.47 = 65.1N

21. A block of wood of rectangular cross-section of sides a and b, and of length L has relative density of S. If the block is to float in water with its longest axis horizontal and the length a vertical, show that for stable equilibrium $\frac{b}{a} > \sqrt{6S(1-S)}$.

Solution:



G = CG, B = Center of buoyancy, M = Metacenter

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For the block to float

Wt. of block = Wt. of water displaced

$\gamma_{block}V_{block} = \gamma_w V_{water\ diplaced}$

$\gamma_w S\ abL = \gamma_w\ byL$

y = Sa

$OB = \frac{Sa}{2}, OG = \frac{a}{2}$

$BG = \frac{a}{2} - \frac{Sa}{2} = \frac{a}{2}(1-S)$

$BM = \frac{I}{V} = \frac{\frac{1}{12}Lb^3}{bLy} = \frac{\frac{1}{12}Lb^3}{bLSa} = \frac{b^2}{12Sa}$

Metacentric height $(GM) = \frac{I}{V} - BG = \frac{b^2}{12Sa} - \frac{a}{2}(1-S)$

For stable equilibrium, GM>0

$\frac{b^2}{12Sa} - \frac{a}{2}(1-S) > 0$

$\frac{b^2}{12Sa} > \frac{a}{2}(1-S)$

$\frac{b}{a} > \sqrt{6S(1-S)}$

22. Consider a homogeneous right circular cylinder of length L, radius R, and specific gravity S, floating in water (S = 1) with its axis vertical. Show that the body is stable is $\frac{R}{L} = [2S(1-S)]^{1/2}$.

Solution:

Length of cylinder = L

Radius of cylinder = R

Specific gravity of cylinder = S

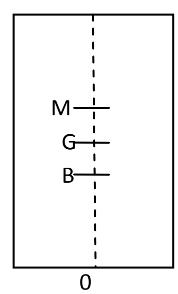
h = depth of immersion

Weight of cylinder = Weight of water displaced

$\gamma_{cyl}V_{cyl} = \gamma_{water}V_{water\ diplaced}$

$Sx9810x\pi xR^2xL = 9810x\pi xR^2xh$

h = SL



Position of center of buoyancy (OB) = SL/2

Position of CG (OG) = L/2

BG = OG-OB = $\frac{L}{2}(1-S)$

$MB = \frac{I}{V} = \frac{\frac{1}{4}x\pi xR^4}{\pi xR^2xh} = \frac{R^2}{4h} = \frac{R^2}{4SL}$

GM = MB-BG = $\frac{R^2}{4SL} - \frac{L}{2}(1-S)$

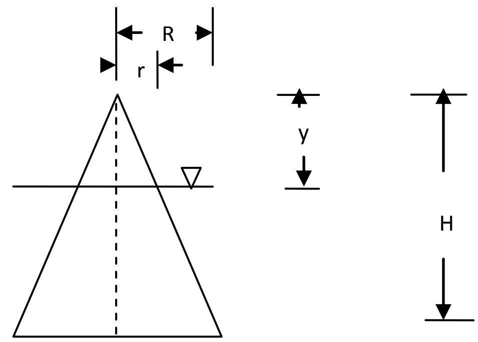
For stable equilibrium, GM $\geq$ 0

$\frac{R^2}{4SL} - \frac{L}{2}(1-S) = 0$

$\frac{R}{L} = [2S(1-S)]^{1/2}$

23. If a solid conical buoy of height H and relative density S floats in water with axis vertical and apex upwards, show that the height above the water surface of the conical buoy is equal to $H(1-S)^{1/3}$.

Solution:
R = Radius of cone
r =Radius of cone at water surface
y = Height above the water surface
H = Height of cone



Weight of cone = Weight of water displaced

$\gamma_{cone} V_{cone} = \gamma_{water} V_{water\ diplaced}$

$Sx\gamma_{water} x \frac{1}{3}\pi R^2 H = \gamma_{water} x \left(\frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 y\right)$

$R^2 SH = R^2 H - r^2 y$

$y = \frac{R^2}{r^2} H(1-S)$ (a)

From similar triangles,

$\frac{R}{r} = \frac{H}{y}$ (b)

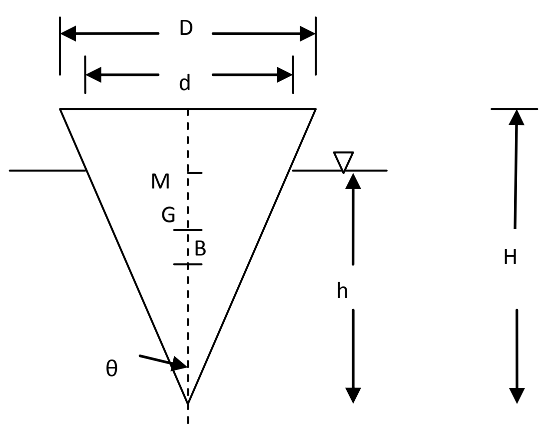
From a and b

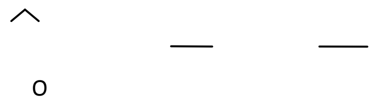
$y = \frac{H^2}{y^2} H(1-S)$

$y = H(1-S)^{1/3}$

24. A cone of base radius R and height H floats in water with the vertex downwards. If θ is the semi-vertex angle of the cone and h is the depth of immersion, show that for stable equilibrium $Sec^2\theta > H/h$.

Solution:
D = Dia. Of cone
d =Dia. Of cone at water surface
h = Depth of immersion
H = Height of cone
2θ = Apex angle
R = Radius of cone
r = radius of cone at water surface





$r = htan\theta$

OG = $\frac{3}{4}H$

OB = $\frac{3}{4}h$

BG = $\frac{3}{4}H - \frac{3}{4}h = 0.75(H-h)$

$MB = \frac{I}{V} = \frac{\frac{1}{4}\pi r^4}{\frac{1}{3}\pi r^2 h} = \frac{0.75r^2}{h} = \frac{0.75(htan\theta)^2}{h} = 0.75htan^2\theta$

GM =$\frac{I}{V} - BG = 0.75htan^2\theta - 0.75(H-h)$

For stable equilibrium, GM>0

$0.75htan^2\theta - 0.75(H-h) > 0$

$htan^2\theta > (H-h)$

$h(1+tan^2\theta) > H$

$Sec^2\theta > H/h$

25. A cone of base diameter d and height H floats in water with the axis vertical and vertex downwards. If the sp gr of the cone material is S, show that for stable equilibrium $H < \frac{1}{2}\left[\frac{d^2 S^{1/3}}{1-S^{1/3}}\right]^{1/2}$.
If S = 0.7, what would be the minimum value of R/H for stable equilibrium?

Solution:

d = Dia. Of cone

d1 =Dia. Of cone at water surface

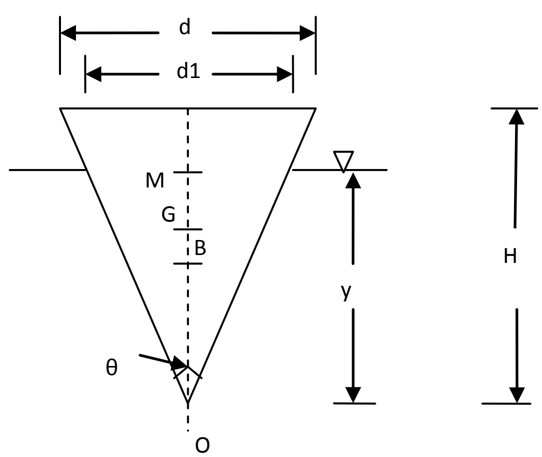
y = Depth of immersion

H = Height of cone

2θ = Apex angle

R = Radius of cone

r = radius of cone at water surface



Weight of cone = Weight of water displaced

$\gamma_{cone}V_{cone} = \gamma_{water}V_{water\ diplaced}$

$Sx\gamma_{water}x\frac{1}{3}\frac{\pi d^2}{4}H = \gamma_{water}x\frac{1}{3}\frac{\pi d1^2}{4}y$

$Sd^2H = d1^2y$ (a)

From similar triangles,

$\frac{R}{r} = \frac{H}{y}$

$\frac{d/2}{d1/2} = \frac{H}{y}$

$d1 = \frac{y}{H}d$ (b)

From a and b

$y = HS^{1/3}$ (c)

OG = $\frac{3}{4}H$

OB = $\frac{3}{4}y$

BG = $\frac{3}{4}H - \frac{3}{4}y = \frac{3}{4}(H-y) = \frac{3}{4}\left(H - HS^{1/3}\right) = \frac{3}{4}H\left(1 - S^{1/3}\right)$

$$MB = \frac{I}{V} = \frac{\frac{1}{64}\pi d1^4}{\frac{1}{3}\frac{\pi d1^2}{4}y} = \frac{3d1^2}{16y} = \frac{3\left(\frac{y}{H}d\right)^2}{16y} = \frac{3d^2y}{16H^2} = \frac{3d^2HS^{1/3}}{16H^2} = \frac{3d^2S^{1/3}}{16H}$$

GM = $\frac{I}{V} - BG = \frac{3d^2S^{1/3}}{16H} - \frac{3d^2S^{1/3}}{16H}$

For stable equilibrium, GM>0

$\frac{3d^2S^{1/3}}{16H} - \frac{3}{4}H\left(1 - S^{1/3}\right) > 0$

$\frac{3d^2S^{1/3}}{16H} > \frac{3}{4}H\left(1 - S^{1/3}\right)$

$H < \frac{1}{2}\left[\frac{d^2S^{1/3}}{1-S^{1/3}}\right]^{1/2}$

Numerical

$H < \frac{1}{2}\left[\frac{d^2S^{1/3}}{1-S^{1/3}}\right]^{1/2}$

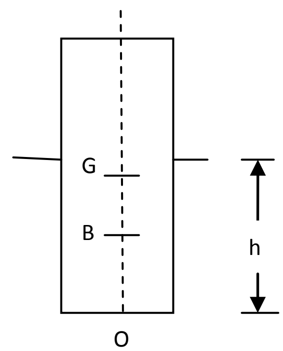
$H < \frac{d}{2}\left[\frac{S^{1/3}}{1-S^{1/3}}\right]^{1/2}$

$H < R\left[\frac{S^{1/3}}{1-S^{1/3}}\right]^{1/2}$

$\frac{R}{H} > \left[\frac{1-S^{1/3}}{S^{1/3}}\right]^{1/2}$

Minimum value of $\frac{R}{H} = \left[\frac{1-S^{1/3}}{S^{1/3}}\right]^{1/2} = \left[\frac{1-0.7^{1/3}}{0.7^{1/3}}\right]^{1/2}$ = 0.355

26. A cylindrical buoy 1.25m in diameter and 1.8m high has a mass of 770kg. Show that it will not float with its axis vertical in sea water of density 1025 kg/m^3. If one end of vertical chain is fastened to the base, find the pull required just to keep the buoy vertical. The CG of the buoy is 0.9m from its base.



Solution:

a) h = depth of immersion

Weight of body (W) = mg = 770x9,81 = 7553.7N

Weight of body = Weight of water displaced

$7553.7 = \rho_{sea\ water} g V_{water\ diplaced}$

$7553.7 = 1025x9.81x\frac{\pi}{4}x1.25^2xh$

h = 0.612m

Position of center of buoyancy (OB) = 0.612/2 = 0.306m

Position of CG (OG) = 1.8/2 = 0.9m

BG = 0.9-0.306 = 0.594m

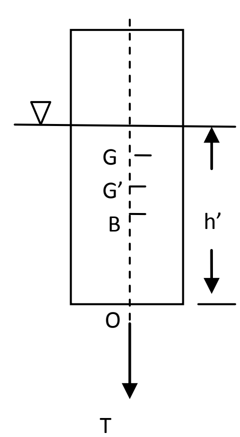
$MB = \frac{I}{V} = \frac{\frac{1}{64}x\pi x1.25^4}{\frac{\pi}{4}x1.25^2x0.612}$ = 0.16m

GM = MB-BG = 0.16-0.594 = -0.434m

As metacentric height GM is negative, the body is unstable and it will not float with its axis vertical.

b) T =pull in chain

Net upthrust (R) = T+W =(T+7553.7)



Finding new depth of immersion (h')

Net upthrust = Weight of water displaced

$R = \rho_{sea\ water} g V_{water\ diplaced}$

$R = 1025x9.81x\frac{\pi}{4}x1.25^2xh'$

h' = 0.00008R

OB = 0.00008R/2 = 0.00004(T+7553.7)

OG = 0.9m

G' = new CG due to T and W (lowered due to T)

$MB = \frac{I}{V} = \frac{\frac{1}{64}x\pi x1.25^4}{\frac{\pi}{4}x1.25^2x0.00008R} = \frac{1220.7}{(T+7553.7)}$

Taking moment about O (W passing through G and R through G')

W OG = R OG'

7553.7x0.9 = (T+7553.7)OG'

$OG' = \frac{6798.33}{T+7553.7}$

BG' = OG'-OB = $\frac{6798.33}{T+7553.7} - 0.00004(T + 7553.7)$

GM = MB-BG' = $\frac{1220.7}{(T+7553.7)} - \frac{6798.33}{T+7553.7} + 0.00004(T + 7553.7)$

$= \frac{-5577.63-0.00004(T+7553.7)^2}{T+7553.7}$

For stable equilibrium, GM>0

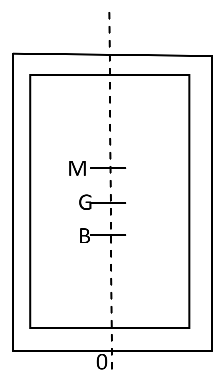
$$\frac{-5577.63 + 0.00004(T + 7553.7)^2}{T + 7553.7} > 0$$

T>4254.8N

27. A hollow cylinder of external radius R and internal radius r, height h and sp. gr. S floats in a liquid of sp. gr. S_0. Show that for its stable equilibrium

$$h \le S_0\sqrt{\frac{R^2 + r^2}{2S(S_0 - S)}}$$

Solution:



Height of cylinder = h

Specific gravity of liquid = S_0

Specific gravity of cylinder = S

h' = depth of immersion

Weight of cylinder = Weight of liquid displaced

$\gamma_{cyl}V_{cyl} = \gamma_{liquid}V_{water\ diplaced}$

$Sx9810x\pi x(R^2 - r^2)xh = 9810xS_0x\pi x(R^2 - r^2)xh'$

$h' = \frac{Sh}{S_0}$

Position of center of buoyancy (OB) = $\frac{Sh}{2S_0}$

Position of CG (OG) = h/2

BG = OG-OB = $\frac{h}{2} - \frac{Sh}{2S_0} = \frac{h}{2S_0}(S_0 - S)$

$MB = \frac{I}{V} = \frac{\frac{1}{4}x\pi x(R^4-r^4)}{\pi x(R^2-r^2)xh'} = \frac{R^2+r^2}{4\frac{Sh}{S_0}} = \frac{S_0(R^2+r^2)}{4Sh}$

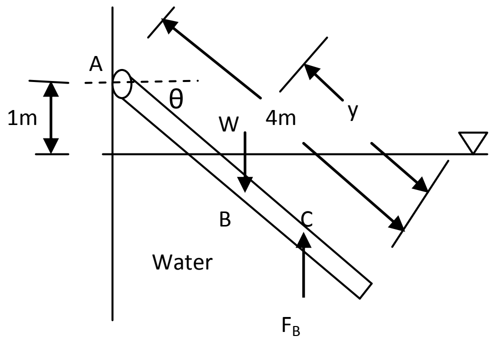
GM = MB-BG $= \frac{S_0(R^2+r^2)}{4Sh} - \frac{h}{2S_0}(S_0 - S)$

For stable equilibrium, GM $\ge$ 0

$\frac{S_0(R^2+r^2)}{4Sh} - \frac{h}{2S_0}(S_0 - S) \ge 0$

$h \le S_0\sqrt{\frac{R^2+r^2}{2S(S_0-S)}}$

28, The wooden beam shown in the figure is 200mmx200mm and 4m long. It is hinged at A and remains in equilibrium at θ with the horizontal. Find the inclination θ. Sp. gr. of wood = 0.6.



Solution:

Length of beam immersed under water = y

Weight of beam (W) = $\gamma_{beam}V_{beam}$ = 0.6x9810x(0.2x0.2x4) = 941.76N

W acts at a distance of 2m from A.

Buoyant force on the beam (F_B) = Weight of water displaced = $\gamma_{water}V_{water\ dispalced}$

= 9810x(0.2x0.2xy) = 392.4y

FB acts at a C.

$AC = 4 - \frac{y}{2}$

Taking moment about hinge A,

$941.76x2cos\theta = 392.4y\left(4 - \frac{y}{2}\right)cos\theta$

$y^2 - 8y + 9.6 = 0$

y = 6.5, 1.47

As y = 6.5 is not possible, y =1.47

$sin\theta = \frac{1}{4-1.47}$

θ = 23.3^0